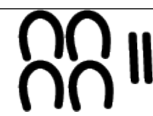
Grade 7/8 Math Circles

February 9, 2022

Ancient Mathematics - Problem Set

1. Complete the chart of values. Some sections may not have an answer.

Hindu-Arabic Numerals	**Roman Numerals**	**Egyptian Hieroglyphs**
3		
		
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2. What counting system do Roman Numerals follow? What about Ancient Egyptians? Are there any aspects about Egyptian Multiplication and Division that remind you of another counting system?

   For a refresher, the lesson on Counting Systems from Fall 2021 can be found here.

3. Evaluate the following using the Egyptian Multiplication technique.
   (a) 17×11
   (b) 61×23
   (c) 256×7

4. Evaluate the following using the Egyptian Division technique.
   (a) $45 \div 18$
   (b) $256 \div 7$
   (c) $1024 \div 30$

5. Answer the following, using Exercise 6 of the lesson as a guide.
   (a) Describe how you could divide 13 loaves of bread among 12 people.

(b) One of the aforementioned 12 people cannot eat bread due to allergies. How could you divide 13 loaves of bread among 11 people?

6. (a) Can you express $\frac{1}{21}$ as a sum of two of the same unit fraction?

   (b) How can we express $\frac{1}{n}$ as a sum of two unit fractions for any positive integer n?

7. As mentioned, we want to find reliable ways to rewrite any fraction as sums of unit fractions. We'll begin by building our intuition of unit fractions. This question shows us that we can rewrite any unit fraction as a sum of two distinct unit fractions.

   (a) Evaluate the following using a method of your choice.

   i. $\frac{1}{3} + \frac{1}{6} =$

   ii. $\frac{1}{4} + \frac{1}{12} =$

   iii. $\frac{1}{5} + \frac{1}{20} =$

   iv. $\frac{1}{6} + \frac{1}{30} =$

   (b) Do you notice a pattern? Can you use this pattern to express $\frac{1}{22}$ as a sum of two unit fractions? (Hint: How do the denominators all relate to each other?)

   (c) In words, describe the generalized pattern for expressing the unit fraction $\frac{1}{n}$ as a sum of two distinct unit fractions. Or, complete the formula given below to describe the pattern algebraically.

$$\frac{1}{n} = \frac{1}{(\qquad)} + \frac{1}{(\qquad)(\qquad)}$$

   (d) Use the formula to express $\frac{1}{2022}$ as a sum of two unit fractions.

8. This question deals with unit fractions with even denominators. Using your findings from the previous question as a guide, complete the following.

   (a) Evaluate the following using a method of your choice.

   i. $\frac{1}{6} + \frac{1}{12} =$

   ii. $\frac{1}{8} + \frac{1}{24} =$

iii. $\dfrac{1}{10} + \dfrac{1}{40} =$

(b) Do you notice a pattern? Can you use this pattern to express $\dfrac{1}{12}$ as a sum of unit fractions?

(c) Complete the formula for expressing the unit fractions $\dfrac{1}{n}$ as a sum of unit fractions where n is an even number. (Hint: It may be helpful to say that $n = 2m$, where m is an integer)

$$\frac{1}{n} = \frac{1}{(\qquad)} + \frac{1}{(\qquad)(\qquad)}$$

9. Recall from the lesson that $\dfrac{2}{3} = \dfrac{1}{3} + \dfrac{1}{3}$ is not a valid expression of an Egyptian fraction (since the two unit fractions aren't distinct).

   (a) Express $\dfrac{2}{3}$ as a sum of three distinct unit fractions. (Hint: Question 7 might be helpful)

   (b) Express $\dfrac{2}{3}$ as a sum of two distinct unit fractions.

10. **Challenge:** Show that we can write $\dfrac{1}{2}$ as a sum of infinitely many unit fractions.